Center-vortex semiclassics on $R^2 x T^2$ and large-N adiabatic continuity

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Goal For 4d confining gauge theories,

Weak-coupling (semiclassical) confinement on $\mathbb{R}^2 \times \underbrace{T^2}_{\text{'t Hooft flux}}$

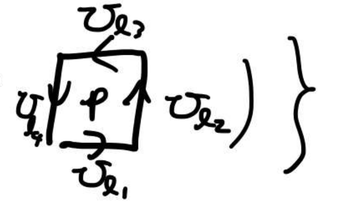
adiabatic continuity $\Longleftrightarrow$

Strong-coupling confinement on $\mathbb{R}^4$

$\mathbb{Z}_N$ 1-form center symmetry & θ-periodicity anomaly

$SU(N)$ link variable; $\mathbb{Z}_N$ plaquette variable; (Background : $dB = 0$)

$$S_{\text{Wilson}}[U_\ell, B_p] = \beta \sum_p \mathrm{Re}\left\{ \mathrm{tr}\left(1 - e^{-\frac{2\pi i}{N} B_p} U_{\ell_3} U_{\ell_4} U_{\ell_2} U_{\ell_1}\right)\right\}$$



B : $\mathbb{Z}_N$-valued 2-form gauge field for center symmetry = 't Hooft flux/twist

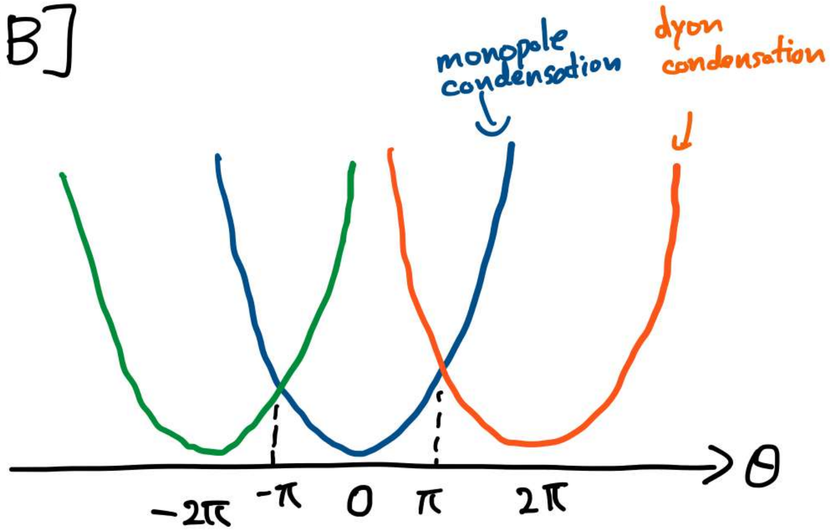
$$\begin{cases} B_p \mapsto B_p + (d\lambda)_p \\ U_\ell \mapsto e^{\frac{2\pi i}{N}\lambda_\ell} U_\ell \end{cases}$$

← gauged center transformation

With the presence of B, θ-periodicity is mildly violated by a contact term of B:

$$Z_{\theta+2\pi}[B] = e^{\frac{2\pi i}{N}\int \frac{1}{2} B \cup B} Z_\theta[B]$$

[Gaiotto, Kapustin, Komargodski, Seiberg 2017
(cf. van Baal '82)
(purely lattice derivation :
Abe, Morikawa, Onoda, Suzuki, YT '23)]



Yang-Mills theory on $\mathbb{R}^2 \times T^2$ & 't Hooft flux

4d SU(N) YM : $\mathbb{Z}_N^{(1)}$ center symmetry

$\mathbb{R}^2 \times T^2 \Longrightarrow$
- $\mathbb{Z}_N^{(1)}$: Area vs Perimeter for 2d Wilson loops
- $\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}$: Conventional center symmetry for Polyakov loops P_3, P_4

Role of 't Hooft flux p

① 4d anomaly is maximally preserved in 2d effective theory

$$Z_{\theta+2\pi}[B] = e^{\frac{2\pi i}{N}\int \frac{1}{2} B \cup B} Z_\theta[B] \xrightarrow{T^2\text{-compact.}} Z_{\theta+2\pi}[B_{2d}] = e^{i\frac{2\pi}{N} p \int B_{2d}} Z_\theta[B_{2d}]$$

② Classical vacuum is unique & $\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}$ symmetric :

$$P_3 P_4 = e^{\frac{2\pi i}{N} p} P_4 P_3 \quad \leftarrow \text{SU(N) clock \& shift algebra}$$

③ Classical vacuum "violates" $\mathbb{Z}_N^{(1)}$ but semiclassically restored as

4d instanton (④) $\xrightarrow{\mathbb{R}^2 \times T^2_{p\text{-twist}}}$ N center-vortex constituents

[Gonzalez-Arroyo, Montero '98]

Adiabatic continuity conjecture

smoothly connected ?

$NL\Lambda \lesssim 1$

- Weakly-coupled
- Many expected features of 4d dynamics are obtained

$L\Lambda \sim O(1)$

- Confinement w/ strongly-coupled dynamics
- Difficult to solve

(cf.
T^4 : twisted Eguchi-Kawai model (Gonzalez-Arroyo, Okawa '83, '10, ...)
$\mathbb{R} \times T^3$: van Baal '80s, Garcia-Perez, Gonzalez-Arroya ... '90s, Yamazaki, Yonekura '17, Cox, Poppite, Wandler '21
$\mathbb{R}^3 \times S^1$: Davies, Hollowood, Khoze '99, Ünsal, Yaffe, Shifman, Poppitz, ... ~'07)

Claim

> Suitable choice of (N, P) achieves the continuity for pure YM w/ $N \gg 1$.

Evidence for $N = 2$ ⇒ Bergner, Gonzalez-Arroyo, Solar 2505.10396

Large-N tachyonic instability ($p=1$)

- 't Hooft flux preserves the $\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}$ symmetric vacuum (= twist eater)

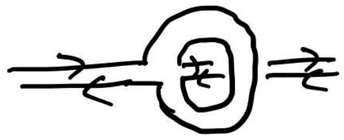
$$P_3 P_4 = e^{\frac{2\pi i}{N}} P_4 P_3 \Rightarrow \begin{cases} P_3 = S \propto \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 1 & & & 0 \end{pmatrix} \\ P_4 = C \propto \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{N-1} \end{pmatrix} \end{cases} \quad \text{(up to gauge trans.)}$$

- Comparison of the classical potential for $\mathbb{R}^2 \times (\underbrace{1\times 1 \text{ lattice}}_{\simeq T^2})$.

$$\begin{cases} S\big|_{\substack{\text{twist eater} \\ (P_3,P_4)=(S,C)}} = 0 \\ S\big|_{\substack{\text{center-broken} \\ P_3, P_4 \propto \mathbb{1}_N}} = \frac{1}{g^2}\left\{ \mathrm{tr}\left(\mathbb{1} - e^{\frac{2\pi i}{N}}\mathbb{1}\right) + \text{c.c.} \right\} \sim O\left(\frac{1}{\lambda_{tH}}\right) \end{cases}$$

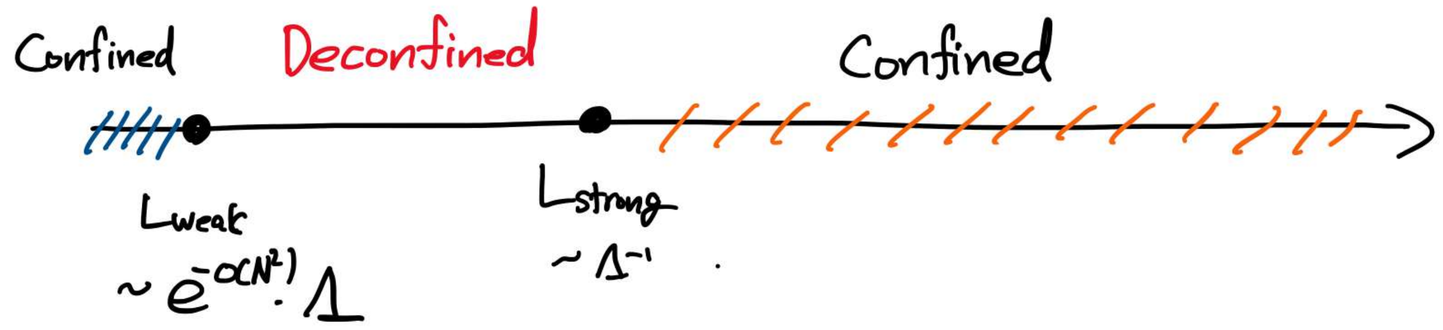
Unless $\lambda_{tH} \lesssim \frac{1}{N^2}$, the twist eater may be destabilized by $O(N^2)$ fluctuations.

- Guralnik, Helling, Landsteiner, Lopez '02 : Tachyonic instability via non-planar self-energy at $\lambda_* \sim N^{-1}$.



- Bietenholz, Nishimura, Susaki, Volkholz '06 : 1st-order deconfinement transition at $\lambda_* \sim N^{-2}$ for $(N=2k+1,\ p=k)$.

Phase diagram for $\mathbb{P}=1$



- Weakly-coupled confinement is separated from strongly-coupled confinement

Two proposals to eliminate the intermediate deconfined phase

We test it for $\mathbb{R}^2\times T^2$.

① Suitably chosen N-dependent twist $\mathbb{P}$ (Gonzalez-Arroyo, Okawa '10)

- Chamizo, Gonzalez-Arroyo : $(N,\mathbb{P})=(F_{k+2},F_k)$ should work (Fibonacci seq.)
  [Analysis is done for 3d YM on $\mathbb{R}\times T^2$]

② Introduce massive adjoint fermions (Azeyanagi, Hanada, Ünsal, Yacoby '12)

Criterion for adiabatic continuity

$$\underbrace{\mathbb{Z}_N^{(1)}}_{\text{center-vortex gas restores it}} \times \underbrace{\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}}_{\text{stabilized for the classical vacuum}} \text{ in } 2d \Longleftarrow \mathbb{Z}_N^{(1)} \text{ in } 4d$$

- Both mechanisms should not be destroyed by $O(N^2)$ quantum fluctuations.

Our interpretation of Chamizo, Gonzalez-Arroyo ('16)

$\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}$ is stabilized against quantum fluctuations for $(N, p) = (F_{k+2}, F_k)$ for $k \gg 1$.

Idea $\mathbb{Z}_N \xrightarrow{N\to\infty} U(1)$ is spontaneously broken to a subgroup $\mathbb{Z}_{n'} \subset U(1)$

if $\frac{p}{N} \xrightarrow{N,p\to\infty} \frac{p'}{n'} \in \mathbb{Q}$.

(Candidate of classical config: [Appendix of our paper.]

$$P_3 = [S_{(n')}^{-p'} \otimes \mathbf{1}_{M\times M}] \oplus S_{(L)}^{-k}$$

$$P_4 = \underbrace{[C_{n'}^{-1} \otimes \mathbf{1}_{M\times M}]}_{n'M\times n'M} \oplus \underbrace{C_{(L)}^{-1}}_{L\times L}$$

w/ $M, L \sim O(N)$)

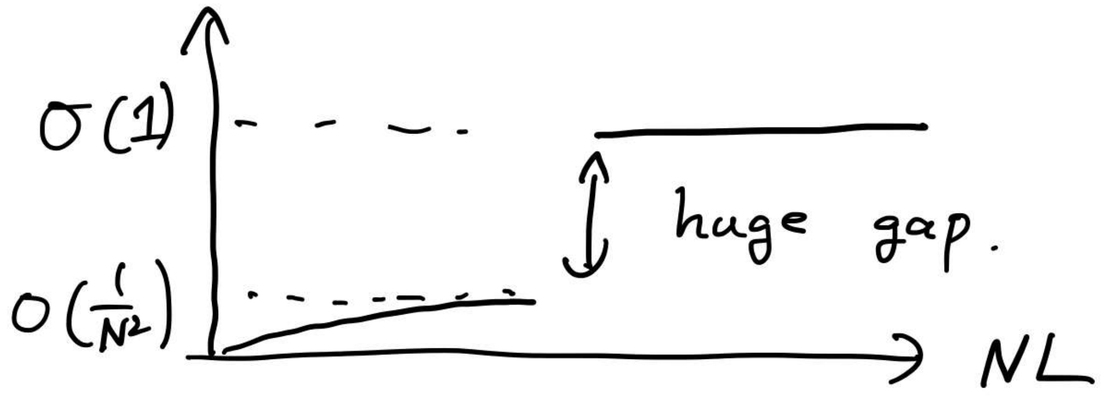
$\Rightarrow \frac{p}{N}$ should converge to an irrational number that is "far from" $\mathbb{Q}$.

Large-N $(\mathbb{Z}_N^{(1)})_{2d}$ breaking for $p=1$

Let us check if $\mathbb{Z}_N^{(1)}$ is also stabilized for $(N,p)=(F_{k+2},F_k)$.

- If $p=1$, dilute gas approximation of center-vortices gives

$$\sigma_{\text{fund}} \sim \Lambda^2 \cdot \underbrace{(NL\Lambda)^{\frac{5}{3}}}_{O(1)\text{ for semiclassics}} \cdot \underbrace{\sin^2\left(\frac{\pi}{N}\right)}_{\to 0 \text{ as } N\to\infty}$$



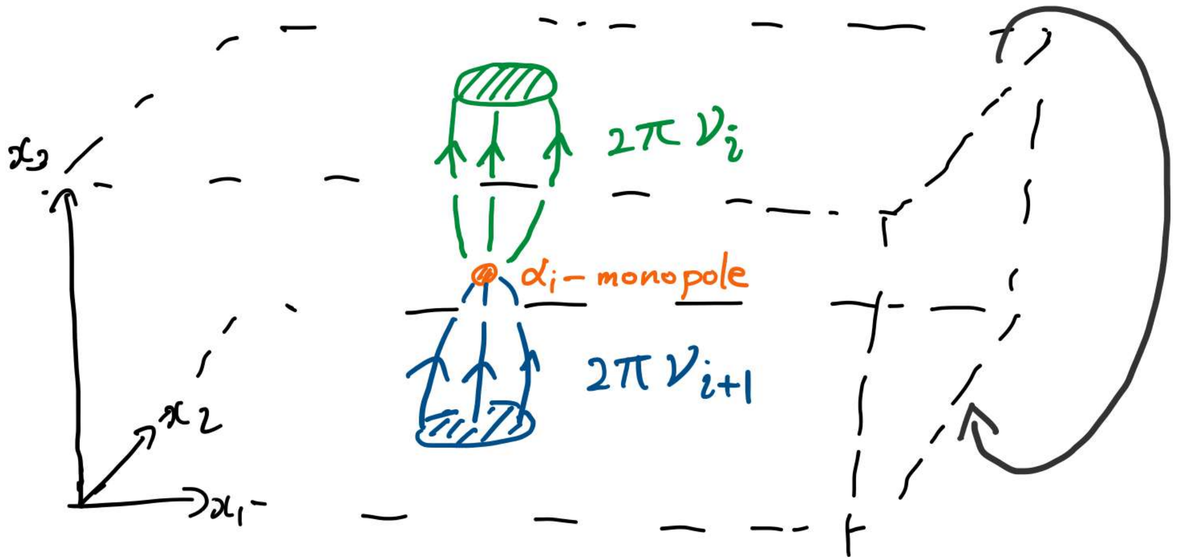
$\Rightarrow$ For suitable p, how can we close this gap?

Center vortex on $\mathbb{R}^2 \times \underbrace{T^2}_{\text{flux}}$ = KuBLLY monopole instanton $(p=1)$

$SU(N)$ gauge field on $\mathbb{R}^3 \times S^1$ w/ nontrivial holonomy : N fundamental monopoles (Lee, Yi '97; Lee, Lu '98; Kraan, van Baal '98)

[$\Rightarrow$ 3d semiclassics by Ünsal, ... since 2007]

α_i – monopole emits the magnetic flux $2\pi\alpha_i = 2\pi[\nu_i - \nu_{i+1}]$.



$\mathbb{Z}_N$-twisted b.c. (= 't Hooft flux on T^2)

$2\pi\nu_i \mapsto 2\pi\nu_{i+1}$

[Hayashi, YT 2405.12402]

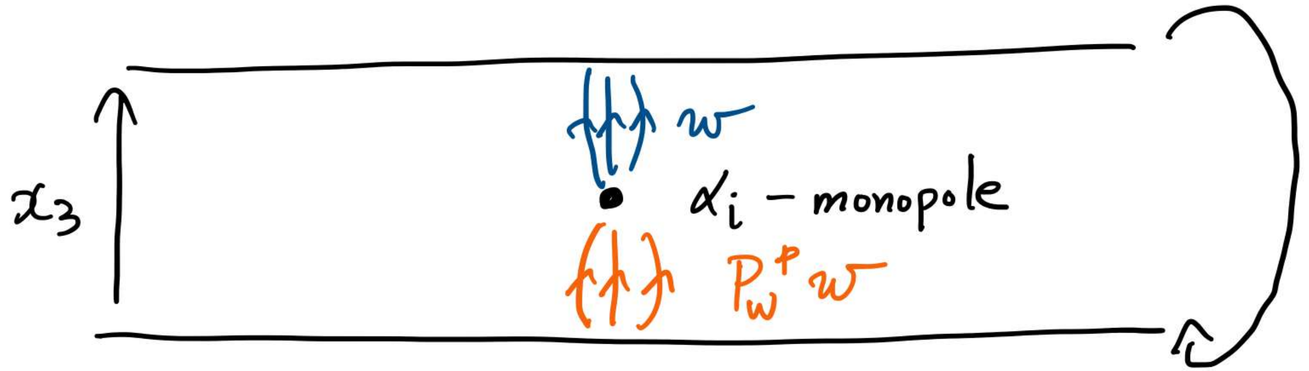
$\mathbb{Z}_N$-twisted b.c. gives the perturbative gap $\frac{2\pi}{NL_3}$ $\Rightarrow$ Magnetic flux localizes.

Monopole = Junction of the center vortex

(cf. Ambjorn, Giedt, Greensite '99, de Forcrand, Pepe '00)

Center-vortex fractional instanton for general (N,p)

Magnetic flux $\vec{w}$ of self-dual center vortex must satisfy



't Hooft twist:

$$\vec{\nu}_i \mapsto \vec{\nu}_{i+\underline{p}} = P_w^p \vec{\nu}_i$$

$$\vec{w} = \vec{\nu}_i + \vec{\nu}_{i+p} + \cdots + \vec{\nu}_{i+(q-1)p}$$

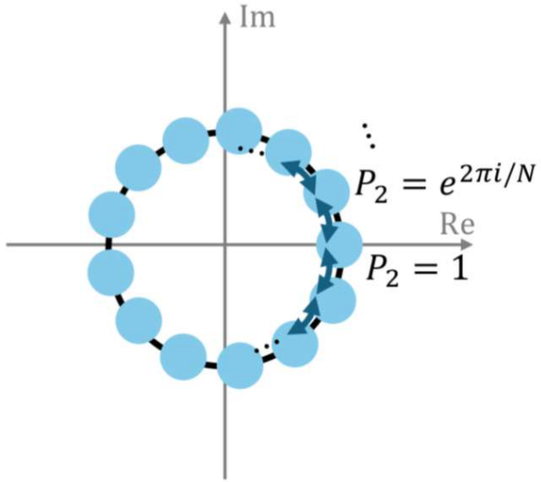
with $q \in \mathbb{Z}_N$ s.t. $q \cdot p = 1 \mod N$

$$-) \; P_w^p \vec{w} = \vec{\nu}_{i+p} + \cdots + \vec{\nu}_{i+(q-1)p} + \vec{\nu}_{i+pq} \qquad (\vec{\nu}_{i+1})$$

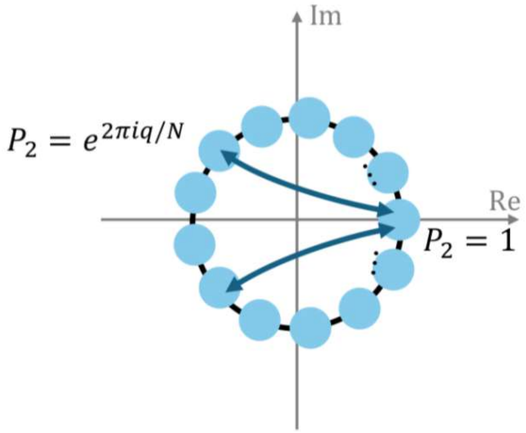
$$\vec{w} - P_w^p \vec{w} = \vec{\nu}_i - \vec{\nu}_{i+1} \; (= \vec{\alpha}_i)$$

$\Rightarrow$ $\frac{1}{N}$-fractional instanton carries $\frac{q}{N}$-flux.

Dilute gap approximation of $\frac{q}{N}$-flux center-vortex instantons for σ_{string}



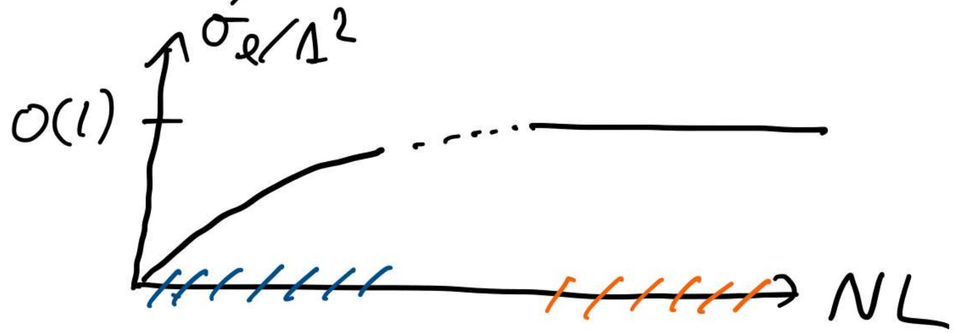
(a) Large N with $p = q = 1$



(b) Large N with $q \sim O(N)$

$$\Rightarrow \sigma_{\ell\text{-string}} \sim \Lambda^2 \underbrace{(NL\Lambda)^{\frac{5}{3}}}_{O(1)} \cdot \sin^2\left(\underbrace{\frac{\pi q}{N}}_{O(1)} \ell\right)$$

For $\ell \sim O(1)$,



Continuity is now reasonable!

Summary

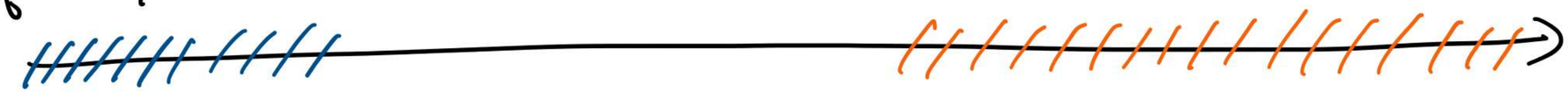
smoothly connected ✓

$NL\Lambda \lesssim 1$

- Weakly-coupled
- Many expected features of 4d dynamics are obtained

$L\Lambda \sim O(1)$

- Confinement w/ strongly-coupled dynamics
- Difficult to solve



if we suitably choose the twist $P(N)$.

We confirmed both

- $\left(\mathbb{Z}_N^{(0)} \times \mathbb{Z}_N^{(0)}\right)_{2d}$ stability
- $\left(\mathbb{Z}_N^{(1)}\right)_{2d}$ stability.

(cf. Gonzalez-Arroyo, Okawa '10
Chamizo, Gonzalez-Arroyo '16, ...)